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ADDITIONAL CONSTRUCTIONS AS A MEANS OF GEOMETRIC PROBLEMS SOLVING

Abstract. The article describes the main additional constructions used in planimetry. And also examples of tasks for a basic and advanced course of the geometry of 7-9 classes are considered for solving where the method of additional construction is used.

Key words: geometry, planimetry, tasks, additional constructions, drawing, auxiliary elements.

Introduction

Geometry is based on solving problems, and solving problems, in turn, is an indispensable means of activating creative abilities among schoolchildren. After all, in order to solve a problem, it is necessary to carry out reasoning based on one’s own experience, knowledge and ability. This knowledge is applied depending on the problem posed and find the correct mathematical result. All this together can be regarded as a learning discovery, which is the best way to develop the student’s creative abilities and his personality as a whole.

As practice shows, while studying a geometry course students have difficulties at the stage of independent search for a solution to problems. This problem is caused by the fact that during teaching geometry much attention is paid to mastering the content of the subject, and the tasks are only used as a reinforcement of the topic studied. Techniques and methods used in solving geometric problems are not analyzed. This leads to the fact that the unconscious and unreasonable solution of the problem does not allow the development of certain rules and criteria by which the student could be guided by the independent solution of problems.

In the psychological, pedagogical and methodical literature, the problem of the inability to independently solve geometric problems is seen in the fact that students are not able to work with a drawing. Drawing in geometry is widely used, and its independent construction really often causes difficulties for students. But at the same time, the drawing is visual support in the conduct of reasoning, serves as a means of clarity, the basis of mental operations and a source of new knowledge. The ability to work with a drawing is very important when studying a geometry course. Construction of the drawing, its transformation expands the role of practical actions in solving problems. As the famous popularizer of science, the mathematician and teacher I.F. Sharygin said, "The main actor of Geometry should be a figure, and the main means of learning should be drawing a picture." [5]

In geometry, there is a huge number of tasks to build, of which of great interest are the tasks of transforming a geometric drawing. This class of tasks is quite complex, examples of such tasks: tasks solved by cutting the shape; tasks to find the geometric location of points; additional construction tasks, etc. When solving this class of problems, the student gets the opportunity to study geometry exactly in the form in which it most corresponds to its real essence, rather than solving the problem using algebraic techniques.

Basic additional constructions in geometry

In our study, much attention is paid to the problem solving with the help of additional constructions. Additional constructions are one of the most geometric methods for solving geometric problems. Therefore, the solution of such problems has a positive effect on the development of the ability to read a drawing. Also, in addition to the ability to read a drawing, the learner develops creative abilities through solving a creative, non-standard task, in this case, a geometric problem that requires additional constructions.

The idea of solving problems using the method of additional constructions is that the drawing to the task is complemented by auxiliary elements. This is necessary in cases where the relationship between the data and the values sought for is difficult to notice, after the addition of the drawing, these connections become more tangible or even obvious.

The choice of this particular method in solving the problem is not always obvious, schoolchildren are more often inclined to solve geometric problems by the so-called traditional method. But there are tasks whose solution is possible only when applying this method. There are also problems in which the application of the method of additional constructions simplifies their solution, or is one of several methods for solving one problem.

Consider the classification [3], which presents the main additional constructions used in solving planimetric problems.

1. Doubling the median of a triangle and completing it to a parallelogram with the centre at the base of this median. (fig. 1).

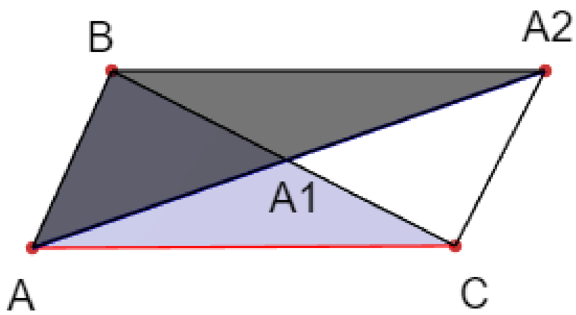


Fig. 1 Illustration for example number 1.

Depending on the content of the task, such extension can be performed for one, two, or even three medians.

2. Suppose that a certain segment (transversal) is given in the triangle, which is drawn through its vertex and is enclosed within this triangle. Then through its base, a ray is drawn into the side of the triangle, parallel to the side, before its intersection with the other side of the triangle (fig. 2).

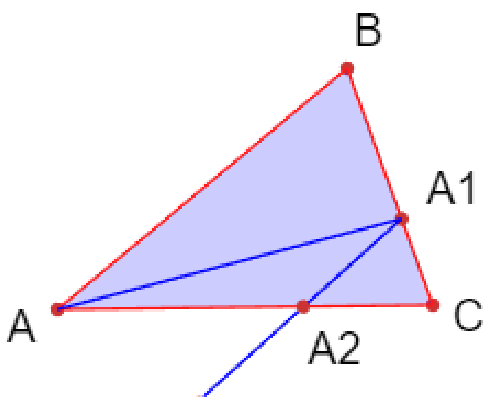


Fig. 2 Illustration for example number 2.

As a result of this additional construction, the situation of Thales arises in the drawing, i.e. parallel lines cut on the secant proportional segments: $\angle ACB$, secant $A_1A_2 \parallel AB$, and we get the following proportional segments:

$$\frac{|CA_3|}{|CA_1|} = \frac{|A_2A|}{|A_1B|}$$

If the segment AA_1 is the median, then the point A_2 is the middle of the side AC.

3. Let the median and some arbitrary segment drawn from another vertex be given in the triangle, it can also be a height, a bisector or a second median. Then through the base of the median inside the triangle, you can draw a beam parallel to this segment, before it intersects with the side of the triangle (fig. 3).

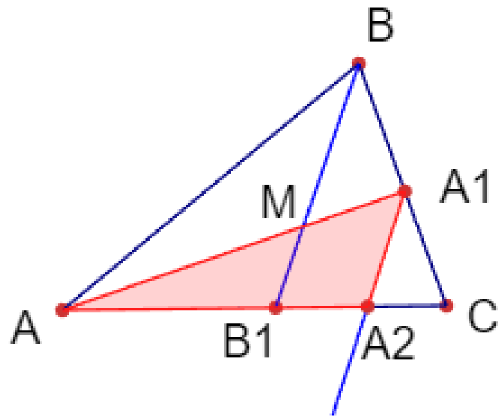


Fig. 3 Illustration for example number 3.

As a result of this additional construction on the drawing, there are two situations of Thales:

1) $\angle ACB$ and secant $A_1A_2 \parallel BB_1$;

2) $\angle A_1AC$ and secant $MB_1 \parallel A_1A_2$.

From the first situation it follows that A_2 is the middle of B_1C, and from the second one we get the relation:

$$\frac{|AM|}{|AB_1|} = \frac{|MA_1|}{|B_1A_2|}$$

4. Let two arbitrary transversals given from different vertices be given in the triangle, then a ray parallel to the other transversal is drawn through the base of one of them to the intersection of the triangle with the side of the triangle (fig. 4).

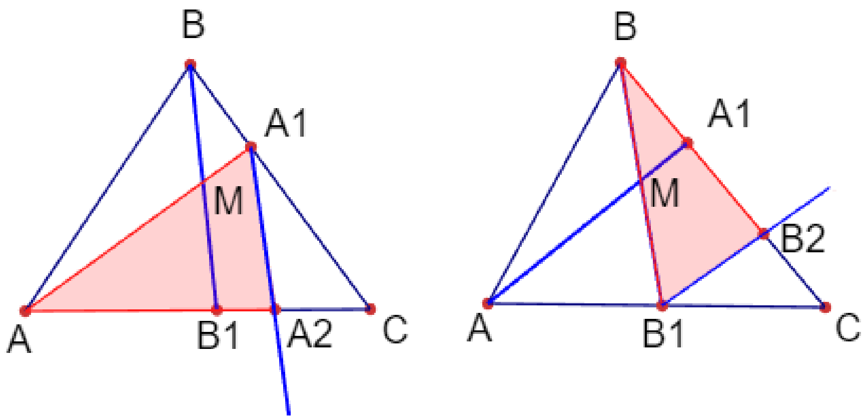


Fig. 4 Illustration for example number 4.

The difference from the previous example is that here the first situation of Thales gives not equal, but proportional segments. Also, you should pay attention to the fact that in this example, and in the two previous ones, similar triangles arise along the way.

5. Let two transversals given from different vertices be given in a triangle, then a straight line parallel to the side of the triangle is drawn through the beginning of one of them (the vertex of the triangle) to the intersection with the continuation of the other transversal (fig. 5).

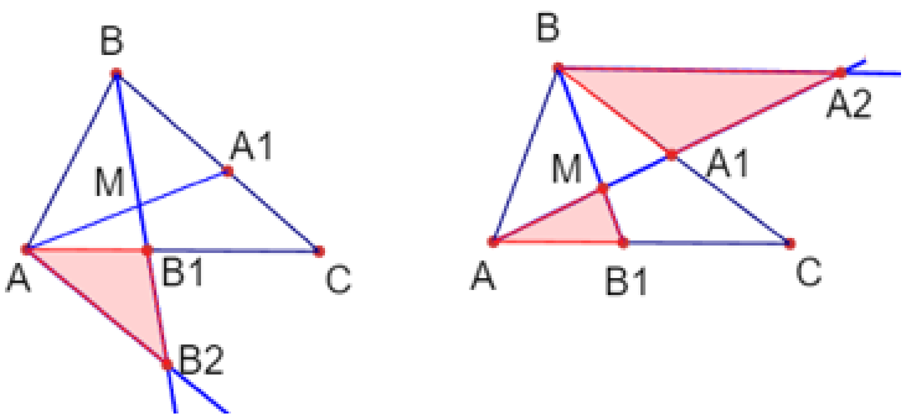


Fig. 5 Illustration for example number 5.

a) $\Delta AMB_2 \sim \Delta A_1MB$ и $\Delta AB_1B_2 \sim \Delta CB_1B$;

b) $\Delta AMB_1 \sim \Delta A_2MB$ и $\Delta AA_1C \sim \Delta A_2A_1B$.

6. Let the right-angled triangle ABC be given, it is completed to an isosceles triangle, in which one of the legs becomes height (median and bisector), and the other half of the base (fig. 6).

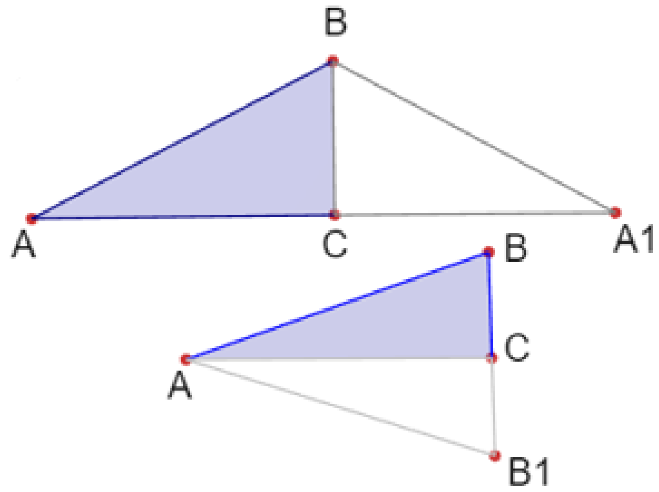


Fig. 6 Illustration for example number 6.

This additional construction leads to the construction of a point symmetrical to the vertex of the acute angle of the triangle with respect to a straight line passing through the leg that does not contain this vertex:

$A_1 = S_{(BC)}(A)$, $B_1 = S_{(AC)}(B)$.

7. If construction is given in which perpendicular straight lines or segments are involved, as well as figures with right angles, then a right-angled triangle is inserted in the drawing, appropriately associated with these elements.

8. Let the trapezoid ABCD be given, then its diagonal BD or side AB is transferred to the vector b, which is equal to one of the bases of the given trapezium (fig. 7).

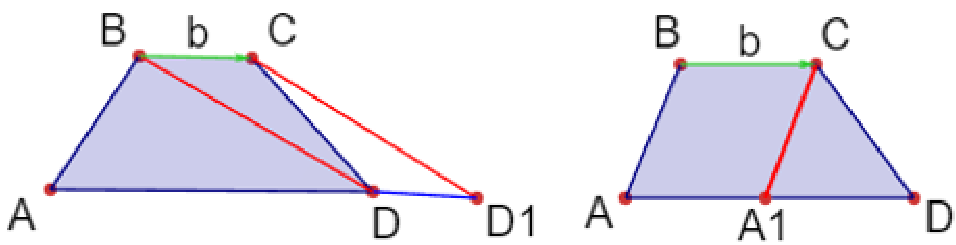


Fig. 7 Illustration for example number 8.

9. If in a triangle, parallelogram or a trapezoid, a bisector of one of the inner corners is specified, then a rhombus is inserted into the drawing, the two sides of which are directed along the sides of the given figure, and this bisector is one of the diagonals (fig. 8-10).

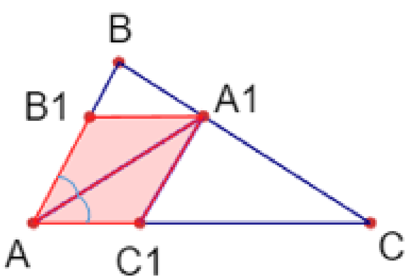


Fig. 8 Illustration for example number 9.

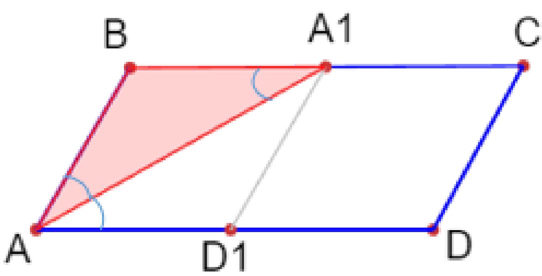
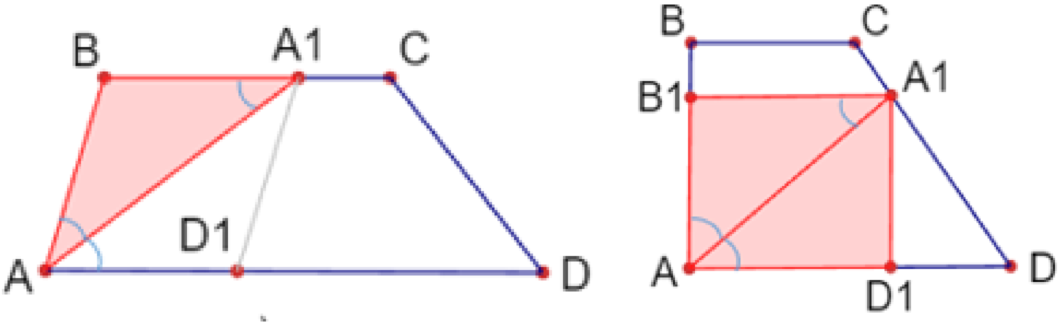


Fig. 9 Illustration for example number 9.

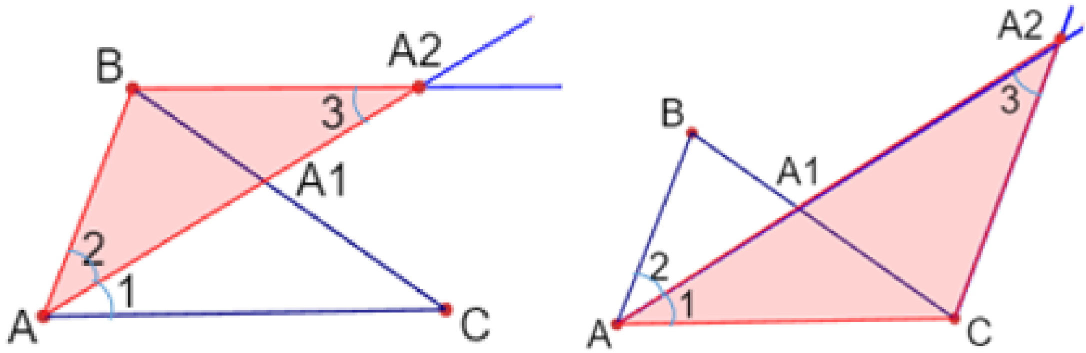


a) *b)*

Fig. 10 Illustration for example number 9.

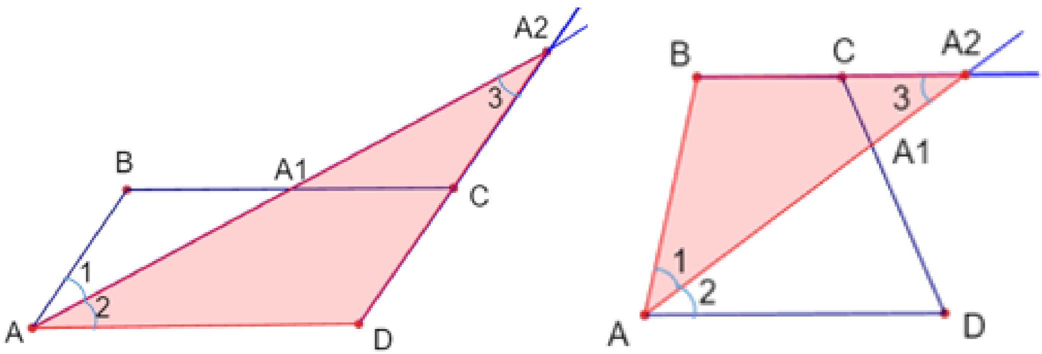
In some cases, it is useful to consider not the whole rhombus, but it's part - an isosceles triangle, one of the sides of which is the side of the given figure, and the basis is the given bisector (fig. 9, fig. 10, a).

10. If a bisector of one of the inner corners is specified in a triangle, parallelogram or a trapezoid, then a triangle is entered into the drawing, one of the sides of which contains this bisector, the second coincides with the side of the original figure, and the third is either parallel to the other side of this figure. continuation (fig. 11–13).



a) *b)*

Fig. 11 Illustration for example number 10.



a) *b)*

Fig. 12 Illustration for example number 10.

To perform such an additional construction, it suffices to continue the bisector to the intersection with the straight line parallel to the side of the given triangle or passing through the side of the given parallelogram and the trapezium.

Note that the constructed triangle is isosceles (its base is the side containing the given bisector). In fact, the angles adjacent to this side are equal, since the angle with the apex at the point of intersection of the indicated lines is crosswise lying with one of the angles into which the bisector divides the angle of the given figure.

11. If the internal rays of these figures (including the diagonal beam) are drawn in the parallelogram or trapezium through two adjacent vertices, then the points of intersection of these rays with the parallel sides of these figures or their extensions are constructed (fig. 13).

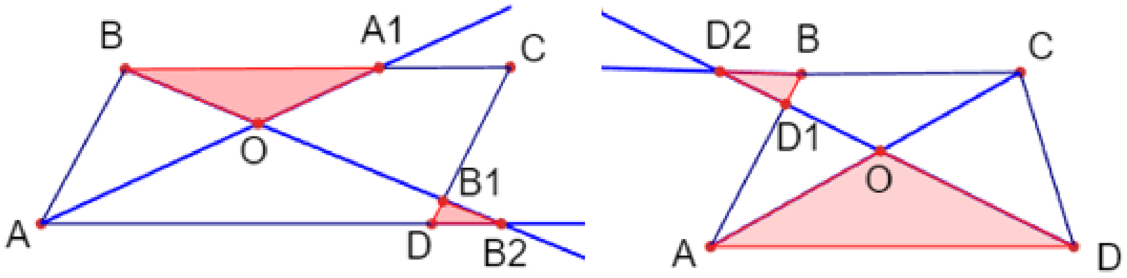


Fig. 13 Illustration for example number 11.

As a result of this additional construction, two pairs of homothetic triangles appear on the drawing. In figure 13, a) the homothetic centres of such triangles are the points O and B_1, and in figure 13, b) the points O and D_1.

12. If a trapezoid is given, then by extending the sides, it is completed to a triangle (fig. 14).

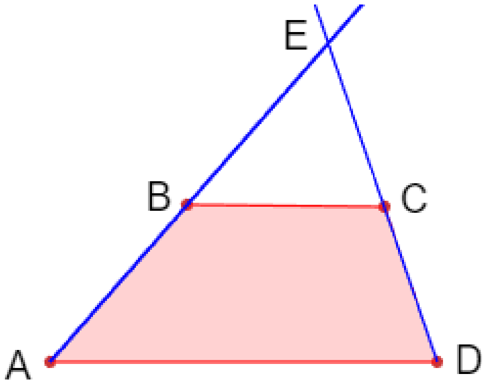


Fig. 14 Illustration for example number 12.

13. If a right triangle is given, then a circle is described around it, the centre of which is the middle of the hypotenuse (fig. 15).

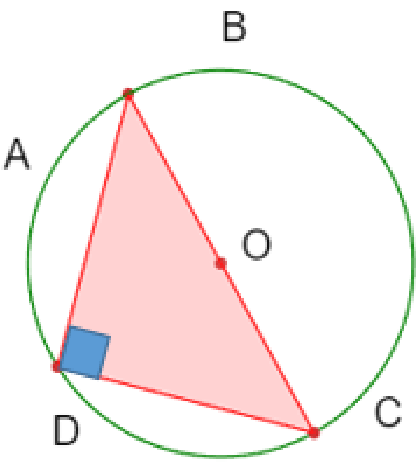


Fig. 16 Illustration for example number 15.

14. If a quadrangle is given, whose sums of opposite angles are equal, then a circle is described around it (fig. 16).

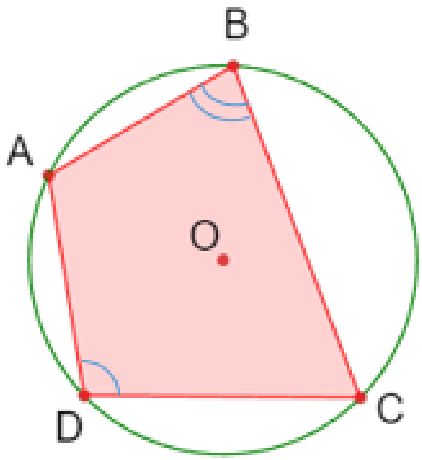


Fig. 16 Illustration for example number 14.

In particular, a square, a rectangle and an isosceles trapezium have a sign of existence for a quadrilateral of a circumscribed circle.

15. If a quadrangle is given, whose sum of opposite sides is equal, then a circle fits into it (fig. 17).

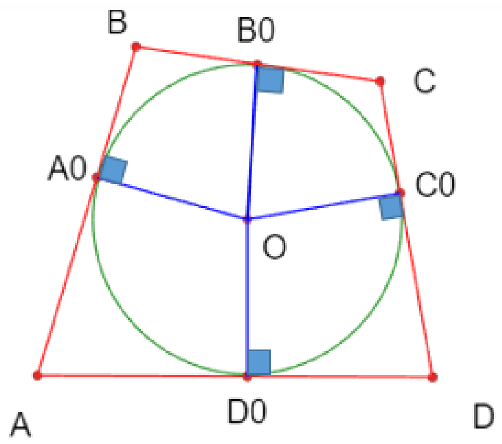


Fig. 17 Illustration for example number 15.

This classification presents the main additional constructions used in the study of planimetry.

Additional constructions in the tasks of the basic school geometry course

The content of mathematics offered for study at each level generally corresponds to the traditional content of school mathematics, which was studied in general education classes and in classes with advanced study of mathematics, respectively. Therefore, the specifics of the chosen learning profile should be reflected in the tasks proposed for solving.

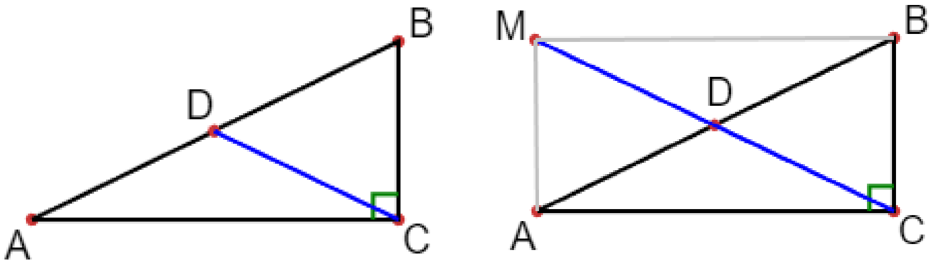
Tasks of a creative, research nature, in the solution of which various qualities of thinking are formed, which often have an impact on the success of future activities, should be studied in both the basic and core levels.

Consider some of the tasks of the school basic geometry course for grades 7-9, which use the method of additional constructions [1, 4].

Problem 1. Prove that the median of a right triangle, conducted to the hypotenuse, is half the hypotenuse.

Decision. Consider a rectangular ΔABC with median CD (fig. 18, a)

1) it is necessary to perform additional construction - doubling the median (CD = DM). Combining the vertices A and B of the triangle ABC with point M, we obtain the AMBC quadrilateral (fig. 18, b).



a) *b)*

Fig. 18 Illustration for problem number 1.

2) it is necessary to prove that AMBC is a rectangle. Consider ∆ADM and ∆CDB: AD = DB, MD = DC (by condition), ∠ADM = ∠CDB (as vertical), it follows that ∆ADM = ∆ADM (according to the first sign of equality of triangles). Similarly, we prove that ∆ADC = ∆MDB and we obtain that AM = CB, MB = AC and ∠C = 90° (by the condition). AMBC is a rectangle.

3) in the rectangle AMBC diagonal AB and MC are equal, it means DC = 1/2AB.

Problem 2. In a right triangle, ABC from perpendicular point M of side AC, perpendicular MH to hypotenuse AB is drawn. Prove that the angles of MHC and MBC are equal.

Decision. Consider the quadrilateral BCMH (fig. 19): Opposite angles are 90°, so the sum of these angles is 180° (∠C + ∠H = 180°). For the proof, it is necessary to perform an additional construction — describe a circle about a quadrilateral (according to the property of the opposite angles of the quadrilateral, the sum of which is equal to 180°).

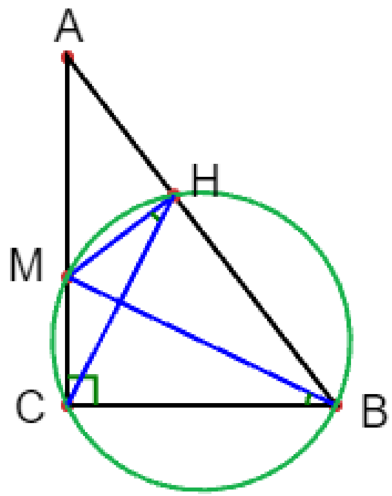


Fig. 19 Illustration for problem number 2.

The inscribed angles MHC and MBC are equal since they are based on the same arc MC.

<u>Additional constructions in the tasks of the advanced school geometry course</u>

The tasks of the advanced school geometry course in high school include tasks of increased complexity. To solve such problems requires ingenuity and geometric intuition. One of the most frequently used methods for solving such problems is the method of additional constructions. If in the basic course of geometry, typical additional constructions are often considered, for which students can be prepared, then non-standard methods of solution can be encountered in tasks of an increased level of complexity. This fact suggests that the ability to solve such problems cannot be developed with the help of concrete examples, such an ability is acquired only with experience.

Problem 1. Inside an isosceles triangle ABC with base BC, a point M is taken such that ∠MBC = 30°, ∠MCB = 10°. Find the angle AMC, if ∠BAC = 80° (fig. 20).

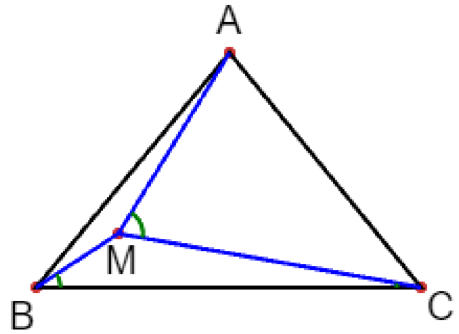


Fig. 20 Illustration for problem number 1.

Decision. ΔABC is isosceles, so the height dropped from the vertex A will be its bisector and median. Perform several additional constructions: we extend the segment BM to the intersection with the height AK at the point E; connect point C and E. Point D is the intersection of AK and CM (fig. 21).

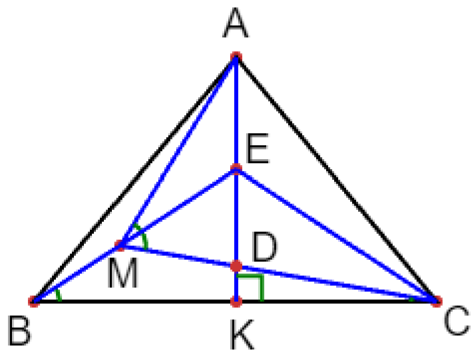


Fig. 21 Illustration for problem number 1

$\angle ABC = \angle ACB = (180°- \angle BAC)/2 = (180°- 80°)/2 = 50°$

ΔBEC – isosceles, it follows: $\angle CEK = \angle BEK = 180° - \angle AKB - \angle EBK = 180° - 90° - 30° = 60°$, means $\angle BEC = 120°$.

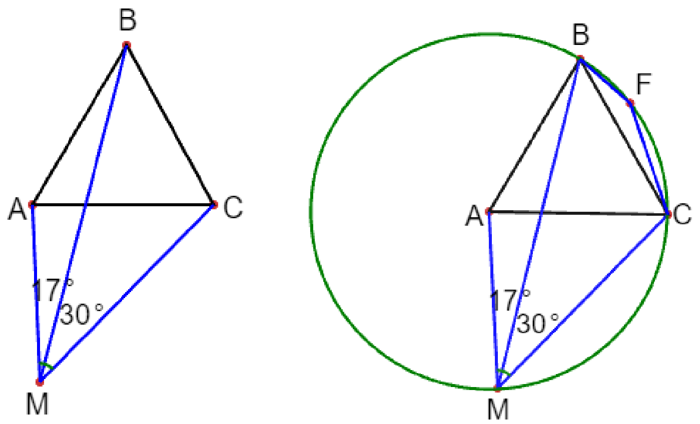
$\angle ECK = \angle EBK = 30°$; $\angle ECM = \angle ECK - \angle MCB = 30°- 10° = 20°$;

$\angle ACE = \angle ACB - \angle BCM - \angle ECM = 50°- 10°- 20°= 20°$;

$\angle CAK = \angle CAB/2 = (80°)/2 = 40°$

Problem 2. Inside the angle ABC of an equilateral triangle ABC, point M is taken so that $\angle BMC = 30°$, $\angle BMA = 17°$ (fig. 22, a). Find the angles BAM and BCM.

Decision. Additional construction is necessary: we draw a circle with radius AB and center at point A. $\angle ABC = 60°$ (ΔABC equilateral), $\angle BMC = 30°$ (by condition). Hence, arc BC = 60°. On $\frown BC$ we take an arbitrary point F (fig. 22, b). $\angle BFC$ relies on the arc BC, $\frown BC = 360° - 60° = 300°$ and BFC = 150°.



a) *b)*

Fig. 22 Illustration for problem number 2.

The sum of the opposite corners of the MBFC quad is 180°. This means that $\angle MBC + \angle BCM = 180°$. Only one circle passes through points B and F with centers at point A and radius AB. Therefore, the point M lies on the circle.

Consider ΔABM: $AB = AM = r$. ΔAMB – isosceles

$$(\angle AMB = \angle ABM = 17°).\ \angle BAM = 180° - (17° + 17°) = 146°.$$

The sum of the angles of the quadrilateral ABCM is 360°.

$$\angle BCM = 360° - 146° - 30° - 17° - 60° = 107°.$$

Methods of educational work at the stage of choosing and drawing additional construction for solving geometric problems and methods of teaching students how to use them

The method of additional construction consists of two types of actions: mental (choice of additional construction) and practical (implementation of additional construction). The ability to solve the problem with the help of additional construction is the ability to find the necessary additional construction and implement it in the drawing.

In order for the student to be able to find the additional construction needed to solve the problem, he should see the need to transform the original drawing, and then from the various types of constructions known to him, choose the one with which you can realize the idea. The student must know various types of additional constructions and possible ways of their implementation. This means that preparatory work in this direction should be carried out with the student.

A student can see the need to transform the original drawing only if he knows how to correctly analyze the condition of the problem and what needs to be found.

Therefore, starting to solve the problem with the help of additional constructions, the student should:

– be able to analyze the requirement and condition of the task;

– find out the types of additional constructions, various ways of their implementation and be able to complete constructions on a geometric drawing.

The implementation of the found additional construction, in some cases, causes the greatest difficulty for students. When implementing additional construction, difficulties most often arise with various possibilities of constructing a new line in the original drawing.

We propose to carry out the development of the ability to solve problems using additional constructions in the following areas.

Preparatory stage.

The preparatory work can be presented in the form of tasks to establish a correspondence between the verbal definition and its geometrical image (depict a figure or a combination of figures in the text; name all possible elements of the figure in the image).

A return to the definition and knowledge of the properties of the geometric figure considered in the problem gives us the opportunity to benefit from this information both in solving problems and in verifying the solution. A mathematical definition creates the meaning of a mathematical term. Using the definition, we establish the relations connecting the newly introduced elements in solving problems by the method of additional construction.

Formative stage.

1. In proving the theorems (by the teacher), it is necessary to justify the construction performed (by students).

2. When solving standard tasks, it is necessary to complete the construction and explain why exactly such a construction was performed in this case, as well as which one is supposed to be the result.

3. When solving problems of any complexity, in which it is necessary to find several solutions. Each method involves the introduction of a new figure (element of the figure). The task of the students also remains to justify the feasibility of constructing a particular figure (or part of a figure).

4. To consolidate the result after solving some problems, the student should list (write out) all completed constructions. Also, at the end of the school year, the student must list (write out) all kinds of additional constructions that they performed in the process of solving various geometric problems for the entire period of study. In this case, it is necessary to analyze and identify which additional constructions most often had to be performed and why.

Conclusion

The complexity of the method of additional constructions lies in the fact that it is often very difficult to understand which additional constructions should be carried out in one or another case. In order to solve problems by this method, it is necessary to solve a sufficiently large number of problems to expand the so-called "geometric outlook". Also, when solving problems using the method of additional constructions, it is necessary to carry out an analysis in order to understand the situations where they are applied.

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