ФИЗИКО-МАТЕМАТИЧЕСКИЕ НАҮКИ

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INTERFERENCE OF X- RAS, MOVING MEDIUMS AND SOURCES

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ИНТЕРФЕРЕНЦИЯ РЕНТГЕНОВСКИХ ЛУЧЕЙ ДВИЖУЩИХСЯ СРЕД И ИСТОЧНИКОВ

Аннотация. Разработаны рентгеновские опыты, анналогичные опытам Саньяка и Майкельсона-Гейля, а также рентгеновский опыт, аналогичный оптическому опыту Физо, которые дают возможность обнаружить влияние движущихся сред и источников на рентгеноинтерференционные картины.

Summary. X - ray experiments analogous to Sagnac and Michelson-Gale experiments as well as the experiment analogous to Fizeau optical experiment enabling to discover the impact of moving mediums and sources on X-ray interference patterns are developed.

Ключевые слова: интерференция, коэффициент увлечения Френеля, рентгеноинтерференционные картины, рентгеновский интерферометр, детектор, резонаторный метод, отражение по Лауэ, отражение по Брэггу.

Keywords: interference, Frenel drag coefficient, X-ray interference patterns, X-ray interferometer, detector, the resonator method, Laue reflection, Bragg reflection.

1. Introduction

The problems on propagation of light in moving mediums with mobile and stationary (regarding the medium) sources are studied both theoretically and experimentally with sufficient accuracy and completeness, its fundamental laws are stated. All these laws and discovered effects also occur in case of X-ray waves (a little difference from X-ray refractive index). As a rule, these effects of moving mediums optics in the range of X-ray wavelength attenuate. For example, the Fresnel drag coefficient

$$\alpha = 1 - \frac{1}{n^2} \tag{1}$$

closely approximates zero. Indeed, as it is known, the X-ray refractive index $n=1-\delta$ slightly differs from zero. The quantity δ (the single decrement of the X ray refractive index) is of the order 10⁻⁵. Consequently, expression (1) with accurate members containing δ^2 can result in $\alpha = -2\delta \sim 2 \cdot 10^{-5}$, while in case of light waves, for instance, for water, it will be $\alpha = 0.434$. Consequently, in translation moving media, the effects due to the drag coefficient for X-rays are much weaker than for light. Moreover, both in forward moving and in rotating media, the optic phenomena conditioned by the medium motion are developed as sharp as the length of the path in these media. The beam path lengthening in case of X-ray waves in rotary mediums is connected with major technical problems: it is necessary to carry out tests in vacuum and make use of specific precision goniometers. Besides, in forward moving media, the length of the beam path is limited by their absorption – the magnification factor of the vacuum equals to point

zero, but the medium absorbs X-rays and limits the length of the path. And, in studying the optic phenomena in rotating media, difficulties relating to the sources of the X-ray rotation arise.

If the mentioned difficulties arise in the X-ray phenomena investigation in the moving media it seems it is impossible to discover the influence of the moving media on the X-ray interference patterns, which are, however, more sensitive to changes in propagation velocities (propagation time) of the waves in moving media. Indeed, the length of the X-ray wave is thousand times less than the length of the optic wave, and therefore the X-ray interference patterns feel the smallest delay of interference waves in regard to each other: one and the same path difference in case of Xrays causes a thousand times more phase difference than in case of light waves.

Under the conditions of the mentioned difficulties, the last circumstance enables to discover the influence of the moving media and the sources on the X-ray interference patterns. If in the X-ray range of wavelengths, experiments similar to the experiments made by Sagnac [1] and Michelson - Gale [2] are realized, they will have essential advantage regarding to the light version of these experiments and they would be miniature gyroscopes which could be located in the artificial Earth satellites and spaceships for measuring their velocities.

The aim of this paper is to develop X-ray experiments analogous to Sagnac and Michelson-Gale experiments, as well as the experiment analogous to Fizeau light experiment.

2. X- ray interference in rotary crystals and sources

The Michelson's negative experiment outcome shows that the forward motion of the Earth cannot be proved by optic (as well as mechanical) experiments. However, this result is not a proof to the abovementioned. It is known that the rotation of the Earth can discovered (measured) both mechanically be (according to experiments analogous to the Fizeau experiment). and optically (according to the experiments analogous to Michelson-Gale experiment).

As it was mentioned above, the realization of such an experiment as Michelson-Gale's for the X-ray range wavelength will give a possibility to study the features of the detector rotation and the source influence on the interference patterns with short waves and develop an X-ray and γ – gyroscope suitable for extraterrestrial measurements.

To realize X-ray experiments, besides the abovementioned, difficulties related to X-ray mirrors arise. The thing is, that mirrors reflect X-rays only under very small sliding angles (under very big angles of incidence), therefore, it is practically impossible to study the influence of rotary mediums on X-ray interference patterns by mirrors. For these purposes the Bragg reflection from atomic planes of monocrystals can be used. However, the realization of sequential Bragg reflections from several crystals, in turn, is connected with problems of alignment. To put the crystals into position of sequential reflections for the same beam is a complex experimental problem which can be solved only by means of specific precision goniometers. In the last case it is difficult to guarantee that the system motion will not violate the crystals alignment of the system towards each other. All these problems can be avoided by using monolithic systems. The reflecting crystals are made of one and the same piece of monocrystal in such a way that they can have a general base (not violating their matrix link and orientation with regard to each other). However, in monolithic samples (systems), the realization of sequential reflections, in particular, cyclic reflections, is a more complex problem. This problem can be solved by using X-ray interferometers and resonators developed at the Chair of Physics of National Polytechnic University of Armenia. We have developed several versions of X-ray experiments in order to study the influence of rotary media on the Xray interference patterns. These versions can be divided into two groups: experiments in which the radiation paths are not locked, and experiments in which the interfering rays make a closed path. We call the experiments of the first group interference, and the experiments of the second group - resonator.

3. Interferometer research on influencing rotary media on the interference patterns of an Xray

Assume we have four monocrystals with planes of reflecting families (hkl) parallel to their major surfaces. These crystals with absolutely the same orientation are located at the vertex of the parallelogram (or rhomb), as it is shown in Fig. 1.

Crystals I and 1V are thin crystals, crystals II and **III** are thick ones. The primary beam **1** in sliding angle θ (Bragg angle) is incident on the first crystal **I**. Part 2 of this beam is reflected from planes (hkl) and falls on crystal II angularly to the Bragg θ . The other part **3** crosses through crystal I in sliding angle θ and is incident on crystal III. Beam 2 completely reflecting from the thick crystal II forms beam 4. The latter angular to the Bragg θ is incident on crystal **IV** where partially reflecting beam 8 is formed, and partially passing beam 6 is formed. Beam 3 completely reflecting from the thick crystal **III** forms beam **5** which sliding angularly θ is incident on the crystal **1V**, where partially reflecting, beam 7 is formed, and partially passing, crystal 9 is formed. Beams 8 and 9 at the plane input (hkl) of the same crystal **IV** interfere with each other; the interference pattern can be registered by detector A, and beams 6 and 7 interfere with each other at the input (\overline{hkl}) of this same crystal. The last interference pattern can be registered by detector **B**. Crystals I and IV should be thin in order for the part of the falling beam on them could reflect, and the other part pass. It is advisable to select the thicknesses of these crystals so that the intensity of the passing and reflecting beams is equal. However, in order for the interference beams (beams 8 and 9, 6 and 7) to be equal to each other (it is necessary to get sharp interference patterns), the crystal thicknesses I and IV must be identical.



Fig. 1. The interferometer for studying the influence of rotary mediums on interference patterns

As it was mentioned above, the interference pattern is formed both between waves 6 and 7, and between waves 8 and 9. However, the pattern contrast range of beams 8 and 9 will be worse, since these waves





Fig. 2. The plant has a rhomb form

Therefore, it is better to carryout measurements on the interference pattern by obtained beams 6 and 7, i.e., by the detector **B** (Fig.1). Angles of the parallelogram are uniquely defined by the Bragg angle θ (Fig. 2). If the parallelogram is monolithic, its sizes are defined by the sizes of the monocrystal of which it is cut. For interferometric research, the most suitable crystals are silicon ones, that is, artificially grown dislocation - free crystals. In this case, the large diagonal of the parallelogram will be not more than 6...7 cm, particularly, the parallelogram can be a rhomb (a = b = c = d) (Fig. 2). To calculate the influence of



Fig. 3. The plant has a quadratic form

rotary mediums on the interference pattern, it is necessary, as a whole, to use the general theory of relativity. However, since in the experiments under study v << c and effects depend on the first order v << c, where v is the velocity of the media, and c is the velocity of light in vacuum, the calculations can be performed just classically, without the relativity theory application [3].

Let us turn to the calculation of the phase difference, arising because of the medium rotation. In case of the stationary plan (Fig. 3), the central angle $\gamma_0 = \pi/2$ corresponds to each side of the rhomb.

39

Rotating with angular velocity $\dot{\omega}$ if the direction of this angular velocity coincides with the direction of the X-ray by passing this angle has the following values:

$$\gamma_{+\omega} = \frac{\pi}{2} + \frac{1}{2}\omega t_+,\tag{2}$$

where t_+ is the time interval, in the course of which the ray from point *A* reaches point *C* in the direction of the medium rotation, i.e. the path *ABC*.

С

For the ray having passed the path in the opposite direction, the expression (2) has the form:

$$\gamma_{-\omega} = \frac{\pi}{2} - \frac{1}{2}\omega t_{-},\tag{3}$$

where t - is the time interval, in the course of which the ray passes from point A to point C on the path ADC. Then paths ABC and ADC are possible to define

by the following ratios:

$$t_{\pm} = 2\sqrt{(AO)^2 + (BO)^2 - 2(AO)(BO)\cos\gamma_{\pm}} = 2\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\gamma_{\pm}},$$
(4)

where $r_1 = AO = OC$, $r_2 = BO = OD$.

When the Bragg angle $\theta = 45^{\circ}$, the setup $r_1 = r_2 = r$ will have a quadratic form, and expression (4) will have the following form:

$$ct_{\pm} = 4r\sin\frac{1}{2}\gamma_{\pm}.$$
 (5)

For difference squares of the bypass time round paths ABC and ADC from (4) we find:

$$t_{+}^{2} - t_{-}^{2} = \frac{8}{c^{2}}r_{1}r_{2}(\cos\gamma_{-} - \cos\gamma_{+})$$

hence:

$$\Delta t = t_{+} - t_{-} = \frac{8r_{1}r_{2}}{c^{2}(t_{+}+t_{-})}(\cos\gamma_{-} - \cos\gamma_{+}).$$
 (6)

Without making a rude mistake, the following approximations can be performed:

$$t_+ + t_- \approx 2t_0, \tag{7}$$

where t_0 is the time for passing X-ray wave paths ABC or ADC when the parallelogram (setup) is at rest:

$$t_0 = \frac{2a}{c} = \frac{2r_1}{c \cdot \cos \theta} \,. \tag{8}$$

With the same accuracy we can assume that

$$\cos \gamma_{-} - \cos \gamma_{+} = 2 \sin \frac{\pi}{2} \sin \left[\frac{1}{4} \omega t_{0}\right] \approx \frac{\omega t_{0}}{2}$$
(9)

Taking into account (7) - (9) expression (6) will have the following form:

$$4l = \frac{2r_1 r_2 \omega}{c},\tag{10}$$

Using (10) we will find the path difference Δl and the phase difference:

$$\Delta l = c \Delta t = \frac{2r_1 r_2 \omega}{c},\tag{11}$$

$$\Delta \phi = \Delta l \frac{2\pi}{\lambda} = \frac{4\pi r_1 r_2 \omega}{c\lambda}.$$
 (12)

Having measured the displacement of interference fringes and the defined phase difference $\Delta \phi$, using (12), it is possible to calculate the angular velocity of the system shown in Figs. 1-3. During these measurements the following problems can appear:

a) because of finite sizes of the sources and the nonmonochromaticyity of the primary beam, the interference pattern obtained may be insufficiently sharp, which makes difficult to observe the shift of fringes owing to the system rotation, and diminishes the accuracy of the phase difference definition $\Delta \phi$;

b) owing to the small wavelength of X-rays, the periods of fringes are small, therefore these fringes are located tough enough; this also makes difficult to observe their shifts;

c) in studying the influence of the Earth rotation on the interference patterns because of the low angular velocity, the effect can be weak and the detection may be difficult.

To overcome these difficulties appropriate measures should be taken. To increase the interference pattern contrast, it is necessary either to monochromatize the primary radiation of the point source by asymmetric reflection (Fig. 4), or to use a circuit with multiple reflections (Fig. 5).

In case of toughly- located interference fringes, it is possible either to use the moire fringes, if they are sharp, or by means of a wedge, to form fringes equal to thicknesses, and the rotation influence of the media can be seen on these fringes (Fig. 6).



At last, if the angular velocity of the Earth rotation results in an insignificant shift of interference fringes, then a diffraction lens can be applied to increase the interference pattern resolution (Fig. 7) [4].



To study the rotary medium influence on X-ray interference patterns, Laue-Laue (LLL) interferometers should also be used (Fig. 8). As a rule, contrasts of interference patterns obtained from LLL interferometers are better than pattern contrasts obtained from interferometers shown in Figs. 1-3, however, in the former case, the light force is low (in case of LLL).

4. The resonator research method of the rotary medium influence on the X-ray interference patterns

In the previous section, interference patterns formed by imposing beams having open trajectories were discussed. We observed the interference pattern in the direction of the primary beam or the first diffracted beam between the waves formed by one-fold or twofold of the Bragg or Laue reflections.



Fig. 9. The scheme of the Sagnac experiment



To lengthen the paths of interfering rays without elongation of setup sizes, it is necessary to close the paths of interfering rays by increasing the number of reflections and observe the interference pattern in the primary point of their formation, that is, to conduct Xray experiments, analogous to Sagnac optic experiments (Fig.9) and Michelson-Gale (Fig. 10) [1,2]. The realization of such experiments is of great difficulty. Indeed, the experiments described in the first section were easy to realize, since the reflection in them came from different sides of the same family of planes, that is, from planes (*hkl*) and (*hkl*). The beam reflected from the planes automatically comes to the position of the planes (*hkl*), therefore it is easy to fabricate monolithic interferometers to study the influence of the rotary mediums on the interference patterns shown in Fig. 1 and 8. There is another problem when we want to close the interference beam paths which are possible to do only by applying X-ray resonators. In this case, it is necessary to develop a monolithic monocrystalline system, ensuring the sequential reflections of propagating two beams in the opposite directions from different (definite) families of blocks of planes in this system, so that the trajectory of these beams is closed in the primary point of their formation.

It seemed that the problem of making such a monolithic system could be solved by using a monolithic resonator. However, along with it, more serious problems arise.



It appears that making of two coherent beams with closed trajectories, propagating in opposite directions and observing their interferences separately is a sufficiently complicated task. To elucidate the character of these difficulties, we will consider some seemingly appropriate schemes shown in Figs. 11 and 12. These schemes are suggested as resonators [5, 6], but, as it is shown in detail in [7], they are not available for these purposes. But as we can see from these

figures, two beams rotating in the opposite beam directions appear in them, and therefore, at first glance, as if these schemes can be used for studying the influence of rotary mediums on interference patterns. The detailed research shows that they are not suitable for these purposes as well. Really, as we can see in these schemes, the interference pattern can be observed only in the OA direction, but, unfortunately, in this direction not only the waves undergone to the rotation

С

E

B

in the system propagate, but the wave reflected in point O without taking part in the rotation as well. It is clear that the last wave is very strong and overlaps the interference pattern obtained as a result of interference waves, taking part in the rotation

For this reason, to study the influence of rotary mediums on the X-ray interference patterns, the scheme shown in Fig. 11 is not applicable. Indeed, the incident beam splits into two beams (crystal O) according to Laue by reflection, and the rotation (inside) takes place by reflections according to Bragg, therefore, the Bragg reflection appears in point O which veils the interference pattern appeared between the rotary waves. Consequently, to study the influence of the rotary media on the resonator patterns by a resonator method, it is necessary to develop a new and more suitable resonator, which was made by us. Its scheme is seen in Fig. 13.



Fig. 13. The resonator for studying the influence of rotary mediums on resonator patterns

The reflection in the first, second and third crystals come from planes $(h_1k_1l_1)$, but the reflection in the fourth crystal comes from planes $(h_2 k_2 l_2)$. In all these crystals, the reflection takes place according to Laue. But as it is seen from the figure, on the first crystal the reflection can also be according to Bragg from planes $(h_2k_2l_2)$ in point A which is absolutely undesirable, because this impedes the observation of the interference patterns, rotating in the paths of beams ObdecO and OcedbO. But this reflection can not even take place. The problem is that the reflection from planes $(h_2k_2l_2)$ in the fourth crystal takes place according to Laue, and on the first crystal, if it ever takes place, it will be according to Bragg. As it is known, in the symmetrical case, the Laue angle of reflections does not differ from the Bragg angle, but the Bragg angle of reflections differs on magnitude.

$$\Delta \theta = \frac{2\delta}{\sin 2\theta}.$$
 (13)

Consequently, if $\Delta \theta$ is greater than the angle area of the Bragg reflection from planes $(h_2k_2l_2)$, then the Bragg reflection in point A from the planes will be absent.

It is clear that the system whose scheme is shown in Fig. 13 will work when satisfying the conditions

$$\theta_1 + \theta_2 = \frac{\pi}{2},\tag{14}$$

where θ_1 and θ_2 are the angles symmetric to Laue reflections respectively from planes $(h_1k_1l_1)$ and $(h_2k_2l_2)$, that is, the Bragg angles of these reflections.

Then the conditions of the Bragg reflection absence in point A from planes $(h_2 k_2 l_2)$ will be

$$\theta_2' - \theta_2 \le \frac{\phi}{2},\tag{15}$$

where ϕ is the angle area width of the Bragg symmetry reflection from planes $(h_2k_2l_2)$.

From conditions (14) and (15), the most unsuitable is the first one, especially for the monolithic crystalline system and for the characteristic radiation (a small set of radiations with different wavelengths). To avoid these difficulties, a non-monolithic system can be used: the fourth crystal should be made from another sample, and this is also undesirable because of the setup alignment and loading with goniometers.

By the way, the Bragg reflections can appear from the surfaces of the second, third and fourth crystals as well (Fig. 13). These reflections do not worsen the resolution (contrast) of the interference patterns to be reflected, but weaken the aperture ratio of the system. Obviously, fulfilling condition (15), these reflections will not appear.

5. X- ray wave propagation in moving mediums

It is interesting to consider a case when the X-ray source and detector are at rest, but the medium in which these rays are propagating is moving, that is, consider an X-ray experiment analogous to Fizeau light experiment. In the case under study, the Fizeau light experiment scheme (Fig. 14) is inapplicable.

Indeed, for X-rays, neither mirrors, nor common lenses are effective, therefore, on these schemes, it is impossible to conduct experiments on studying the influence of the moving medium on the X-ray interference patterns.



Fig. 16. X- ray experiment analogous to Fizeau light experiment



Fig. 14. Fizeau experiment on the Fresnel magnification factor definition

Fig. 15. A four- crystal X-ray interferometer

Obviously, the most suitable for our purposes is the application of the developed by us a four-crystal Xray interferometer [8,9], one of the versions is shown in Fig, 15. This interferometer differs from a common three-crystal interferometer (Fig. 8) by not only a great resolution, but also by the fact that the rays in it, propagating between the second and third blocks parallel to each other converge into one point on the fourth block: rays 1 and 2 converge into point A, and rays 3 and 4 converge into point B (Fig. 15). Just these parallel parts of the rays (between the second and third blocks) can be used for studying the influence of the moving media on the X-ray interference pattern as it is shown in Fig. 16. The primary beam 1 in the Bragg angle falls on crystal **1**, from there its part is reflected (beam **2**), the other part passes (beam **3**). Beam **3** incidents crystal **II** and reflecting from it (beam **4**) through the medium l and crystal **III** falls into point B of crystal **1V**. Beam **2** passing through crystal **II** and medium l falls on crystal **III** and reflecting from it (beam **5**) falls on point B of crystal **IV**.

In a stationary liquid (water) in the vessel l in the point B, a definite interference pattern appears. When the liquid moves, this pattern changes, because the fluid velocity direction in the upper vessel is opposite to the X-ray wave propagation velocity, but in the lower vessel, these velocities coincide in directions, therefore the phase difference appears between the interfering

waves in point *B*, that is, the shift of the interference fringes taks place. The phase difference between the interfering waves in point *B* can be specified as follows: in the ideal interferometer design and stationary liquid, the phase difference equals zero. The phase difference between these waves appears only because of the fluid motion. The wave delay time whose direction is opposite to the direction of the water flow towards the wave, propagating in the direction of the flow can be defined with the magnification factor.

$$\Delta t = \frac{l}{\frac{c}{n} - v\left(1 - \frac{1}{n^2}\right)} - \frac{l}{\frac{c}{n} + v\left(1 - \frac{1}{n^2}\right)}.$$

This expression with accuracy of members containing a multiplier $(2\delta^2)$ can be reduced to

$$\Delta t = -\frac{4lv\delta}{c^2},$$

where for path difference Δl and phase difference $\Delta \phi$, we obtain the following expressions:

$$\Delta l = c \Delta t = -\frac{4 l v \delta}{c}, \tag{16}$$

$$\Delta \phi = \Delta l \frac{2\pi}{\lambda} = -\frac{8\pi l v \delta}{c \lambda}.$$
 (17)

Having specified the phase difference from the experiment, it is possible to calculate the magnification factor by expression (17).

Conclusions. X- ray physics experiments were developed analogous to the Sagnak and Michelson -Gale experiments, as well as the optical range Fizeau experiment, permitting an observation of the influence of moving media and sources on X- ray interference patterns.

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ПРЕЦЕССИЯ И НУТАЦИЯ УПРУГОЙ ЗЕМЛИ

Ключевые слова: кинетический момент, система отсчета, плоскость эклиптики, прецессия, нутация, момент центробежных сил, нутация Чендлера.

1. Непредсказуемость некоторых гармоник вращения Земли

В докладе «Нутация неупругой Земли» (2003 ведущих сотрудников г.) олного ИЗ международного коллектива по разработке Европейской навигационной системы GALILEO дра В.Е. Жарова отмечается, что в изучении этого вопроса достигнуты значительные успехи. Однако существует гармоника, так называемая, почти суточная нутация, которая вносит наибольшую погрешность в теоретические расчеты. Генеральная Ассамблея Международного астрономического союза (MAC) настоятельно рекомендовала обратить особое внимание на исследования непредсказуемой почти суточной нутации, и даже указывала, что решение проблемы необходимо искать в рамках теории гироскопа [2].

Происхождение данной гармоники неизвестно, однако теория гироскопа, исходя из некоторых начальных измерений, позволяет произвести расчеты. Приведем выдержку из наиболее часто встречающейся трактовки, оставив

свою нумерацию: «Полюс мира, который определяет ось вращения, немного отличается от геометрического полюса, который лежит на оси симметрии... С помошью астрономических измерений можно определить относительную скорость полюса, которая по теории равна

$$p = \frac{C-A}{A}n,\tag{1}$$

где п -угловая скорости собственного вращения

[А и С – экваториальный и угловой моменты инерции (обозначения из оригинала)].

Наблюдения, проведенные между 1890 и 1895 годами, показали, что величина периода, по определению Чэндлера, была равна около 428 дням [подтверждается современными наблюдениями]. Было также показано, что угол между осями имеет порядок 0,1", что соответствует около 4 м на полюсе. Однако другие наблюдения [расчеты] показали, что моменты инерции Земли имеют соотношение